201-NYB-05 - Calculus 2
REVIEW WORKSHEET FOR TEST #3

1. Find the general term of the following sequence, determine if it converges, and if so to what limit.

$$\frac{2}{1},\quad \frac{3}{3},\quad \frac{4}{5},\quad \frac{5}{7},\quad \ldots$$

2. Determine the convergence or divergence of the **sequences** given by the following general term a_n.

(a) $1+2\left(\frac{4}{5}\right)^n$ (b) $\frac{\ln(3/n^2)}{\ln(1/n)}$ (c) $\frac{2(-1)^n\sin(n^2)}{n+\ln(n)}$

3. Determine whether each **series** is convergent or divergent. If the series is convergent, find the sum.

(a) $\sum_{n=1}^{\infty}\frac{(-1)^n-e^{n+1}}{3^n}$

(b) $6+3+1.5+0.75+\ldots$

(c) $\sum_{n=1}^{\infty}\left(\frac{2}{3}+\frac{2}{5}\right)^n$

(d) $\sum_{n=1}^{\infty}\arctan(n+1)-\arctan(n)$

(e) $\sum_{n=1}^{\infty}\frac{4}{4n^2-1}$

4. Show that the series $\sum_{n=1}^{\infty}e^{-n}$ is convergent in **four different ways**.

5. Determine whether each **series** is convergent or divergent.

(a) $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$

(b) $\sum_{n=3}^{\infty}\frac{n^3-5n}{2n^5+n^4}$

(c) $\sum_{n=1}^{\infty}\frac{n!}{n3^n}$

(d) $\sum_{n=1}^{\infty}\frac{1}{(n+1)\sqrt{\ln(n+1)}}$

(e) $\sum_{n=1}^{\infty}\frac{n}{(n+1)3^{n+1}}$

(f) $\sum_{n=1}^{\infty}\left(\frac{3n}{2n+3}\right)^n$

(g) $\sum_{n=1}^{\infty}\frac{n!}{1\cdot3\cdot5\cdot7\cdots(2n-1)\cdot(2n+1)}$

(h) $\sum_{n=1}^{\infty}\frac{5^n}{4^n-n}$

(i) $\sum_{n=0}^{\infty}\frac{2^n\arctan(n)^n}{e^n}$

(j) $\sum_{n=1}^{\infty}\left(1-\frac{2}{n^2}\right)$

(k) $\sum_{n=0}^{\infty}\frac{\sqrt{n+2}}{\sqrt{n^2+1}}$

(l) $\sum_{n=1}^{\infty}\tan(2^{-n})$

(m) $\sum_{n=2}^{\infty}\tan\left(\frac{\pi n^2+n}{1+4n^2}\right)$

(n) $\sum_{n=1}^{\infty}\frac{3}{n\sqrt{n}}$

6. For each alternating series below, determine if it is divergent, conditionally convergent or absolutely convergent.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (n+1)!}{2^{4n}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{\sqrt[3]{n+1}}$

(b) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{-1}{\ln(n)}\right)^n$ (d) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{n}{n^3+2n^2}$

7. Find the interval and radius of convergence for each power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3n+1}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{3^{n+1}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (n+2)! x^n}{n!}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n (x+3)^n}{n}$

8. Find the MacLaurin series for the function $f(x) = \dfrac{1}{2+3x}$. Then, determine the interval and radius of convergence for the resulting series.

9. Find the first four nonzero terms in the Taylor series for the given function around the specified point.

(a) $f(x) = \sqrt{x}$ around $x = 4$ (b) $f(x) = \cos(\pi x)$ around $x = 1$

10. Suppose a_n is a positive, decreasing sequence such that the series $\sum a_n$ is convergent. Prove that the following series are also convergent.

(a) $\displaystyle\sum \frac{a_n}{1+a_n}$ (b) $\sum (a_n - \sin(a_n))$ (c) $\sum (-1)^n \tan(a_n)$

ANSWERS:

1. $a_n = \dfrac{n+1}{2n-1}$, converges to $\dfrac{1}{2}$

2. (a) converges to 1 (b) converges to 2 (c) converges to 0 (squeeze theorem)

3. (a) convergent, sum $= -\dfrac{1}{4} - \dfrac{e^2}{3-e}$ (b) convergent, sum $= 12$ (c) divergent
   (d) convergent, sum $= \pi/4$ (e) convergent, sum $= 2$

4. four ways: geometric series, integral test, ratio test and root test

5. The tests suggested below may not be the only way to do it!
   (a) C (ratio)
   (b) C (CT or LCT)
   (c) D (ratio)
   (d) D (IT)
   (e) C (ratio or LCT)
   (f) D (root or DT)
   (g) C (ratio)
   (h) D (CT)
   (i) D (root)
   (j) D (DT)
   (k) D (LCT)
   (l) C (LCT with $b_n = 2^{-n}$)
   (m) D (DT)
   (n) C (p-series)

6. (a) D (ratio) (b) AC (root) (c) CC (LCT + AST) (d) AC (CT)

7. (a) $[-1,1)$, $R=1$ (b) $(-1,1)$, $R=1$ (c) $(-\sqrt{3},\sqrt{3})$, $R=\sqrt{3}$ (d) $(-7/2,-5/2)$, $R=1/2$

8. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{2^{n+1}} x^n$ interval $= (-2/3, 2/3)$, $R = 2/3$

9. (a) $2 + \dfrac{1}{4}(x-4) - \dfrac{1}{64}(x-4)^2 + \dfrac{1}{512}(x-4)^3 + \ldots$

   (b) $-1 + \dfrac{\pi^2}{2}(x-1)^2 - \dfrac{\pi^4}{24}(x-1)^4 + \dfrac{\pi^6}{720}(x-1)^6 - \ldots$

10. (a) CT (b) LCT (c) AST