Ratio Test

2014 BC6

6. The Taylor series for a function f about x = 1 is given by $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n}(x-1)^n$ and converges to f(x) for $|x-1|<R$, where r is the radius of convergence of the Taylor series.

a) Find the value of R.

2009 BC6

The Maclaurin series for e^x is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \cdots \frac{x^n}{n!} + \cdots$. The continuous function f is defined by $f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \neq 1$ and $f(1) = 1$. The function has derivatives of all orders x = 1.

Use the ratio test to find the interval of convergence for the Taylor Series given $1 + \frac{(x-1)^2}{2} + \frac{(x-1)^4}{6} + \frac{(x-1)^6}{24} + \cdots + \frac{(x-1)^{2n-2}}{n!}$.

2012 #6

The function g has derivatives of all orders, and the Maclaurin series for g is $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots$

a) Using the ratio test, determine the interval of convergence of the Macluarin series for g.

2011 BC6 Form B

The Maclaurin series for ln(1 + x^3) is $x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \cdots + (-1)^n \frac{x^{3n}}{n} + \cdots$.

a. The radius of convergence of the Maclaurin series for f is 1. Determine the interval of convergence. Show the work that leads to your answer.

2010 BC6 Form B

The Maclaurin series for the function f is given by $f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}$ on its interval of convergence.

a) Find the interval of convergence for the Maclaurin series of f. Justify your answer.

2009 BC6 Form B

The function f is defined by the power series

$$f(x) = 1 + (x+1) + (x+1)^2 + \cdots + (x+1)^n + \cdots = \sum_{n=0}^{\infty} (x+1)^n$$

a) Find the interval of convergence of the power series for f. Justify your answer.
b) The power series above is the Taylor series for f about x = -1. Find the sum of the series for f.

2006 BC6

The function f is defined by the power series $f(x)=\frac{-x}{2}+\frac{2x^2}{3}-\frac{3x^3}{4}+\cdots+\frac{(-1)^n nx^n}{n+1}$ for all real numbers x for which the series converges.

The function g is defined by the power series $g(x)=\frac{-x}{2!}+\frac{{}^{2}}{4!}-\frac{x^3}{6!}+\cdots+\frac{(-1)^n x^n}{(2n)!}$ for all real numbers x for which the series converges.

a) Find the interval of convergence of the power series for f. Justify your answer.

b) Find the interval of convergence of the power series for g. Justify your answer.

14. The sum of the infinite geometric series $\frac{3}{2}+\frac{9}{16}+\frac{27}{128}+\frac{81}{1024}+\cdots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

6. What are all values of p for which $\int_1^{\infty}\frac{1}{x^{2p}}\,dx$ converges?

A) $p < -1$ B) $p > 0$ C) $p > ½$
D) $p > 1$ E) There are no values of p for which this integral converges

4. Consider the series $\sum_{n=1}^{\infty}\frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

$A)\ \lim_{n\to\infty}\frac{e}{n!}<1$ $B)\ \lim_{n\to\infty}\frac{n!}{e}<1$ $C)\ \lim_{n\to\infty}\frac{n+1}{e}<1$

$D)\ \lim_{n\to\infty}\frac{e}{n+1}<1$ $E)\ \lim_{n\to\infty}\frac{e}{(n+1)!}<1$

12. Which of the following series converges for all real numbers of x?

$A)\ \sum_{n=1}^{\infty} \frac{x^n}{n}$ $B)\ \sum_{n=1}^{\infty} \frac{x^n}{n^2}$ $C)\ \sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ $D)\ \sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$ $E)\ \sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

15. What are all values of x for which the series $\sum_{n=1}^{\infty} \left(\frac{2}{x^2+1}\right)^n$ converges?

A) -1 < x < 1 B) x > 1 only C) $x \geq 1$ only

E) x < - 1 and x > 1 only F) $x \leq -1$ and $x \geq 1$

79. Le f be a positive, continuous, decreasing function such that $a_n = f(n)$.
If $\sum_{n=1}^{\infty} a_n$ converges to k, which of the following must be true?

$A)\ \lim_{n\to\infty} a_n = k$

$B)\ \int_1^n f(x)dx$ diverges

$C)\ \int_1^{\infty} f(x)dx$ diverges

$D)\ \int_1^{\infty} f(x)dx$ converges

$E)\ \int_1^{\infty} f(x)dx = k$

82. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, which of the following statements must be true?

$A)\ \sum_{n=1}^{\infty} (-1)^n a_n$ converges

$B)\ \sum_{n=1}^{\infty} (-1)^n b_n$ converges

$C)\ \sum_{n=1}^{\infty} (-1)^n b_n$ diverges

$D)\ \sum_{n=1}^{\infty} b_n$ converges

$E)\ \sum_{n=1}^{\infty} b_n$ diverges

22. If $\lim_{b\to\infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

$(A)\ \sum_{n=1}^{\infty} \frac{1}{n^p}$ converges $(B)\ \sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges $(C)\ \sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

$(D)\ \sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges $(E)\ \sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

18. Which of the following series converge?

$I.\ \sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None (B) II only (C) III only (D) I and II only (E) I and III only

84. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

20. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converges?

(A) $-3 \le x \le 3$ (B) $-3 < x < 3$ (C) $-1 < x \le 5$ (D) $-1 \le x \le 5$ (E) $-1 \le x < 5$

24. Which of the following series diverge?

$I.\ \sum_{n=0}^{\infty} \frac{\sin 2}{\pi}$

$II.\ \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

$III.\ \sum_{n=1}^{\infty} \frac{e^n}{e^n+1}$

A) III only B) I and II only C) I and III only
D) II and III only E) I, II, and III

22. What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p+1}$ converges?

A) $p > 0$ B) $p \ge 1$ C) $p > 1$ D) $p \ge 2$ E) $p > 2$