Ratio Test

2014 BC6

6. The Taylor series for a function f about x = 1 is given by \[ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n} (x - 1)^n \] and converges to f(x) for \(|x - 1| < R\), where r is the radius of convergence of the Taylor series.

a) Find the value of R.

2009 BC6

The Maclaurin series for \(e^x\) is \(e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots\). The continuous function f is defined by \(f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}\) for \(x \neq 1\) and \(f(1) = 1\). The function has derivatives of all orders at \(x = 1\).

Use the ratio test to find the interval of convergence for the Taylor Series given
\[ 1 + \frac{(x-1)^2}{2} + \frac{(x-1)^4}{6} + \frac{(x-1)^6}{24} + \cdots + \frac{(x-1)^{2n-2}}{n!}. \]

2012 #6

The function g has derivatives of all orders, and the Maclaurin series for g is
\[ \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots \]

a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.
The Maclaurin series for \( \ln(1 + x^3) \) is
\[
\sum_{n=1}^{\infty} \frac{(-1)^n x^{3n}}{n} = x - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \cdots.
\]

a. The radius of convergence of the Maclaurin series for \( f \) is 1. Determine the interval of convergence. Show the work that leads to your answer.

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The Maclaurin series for the function \( f \) is given by
\[
f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1}
\]
on its interval of convergence.

a) Find the interval of convergence for the Maclaurin series of \( f \). Justify your answer.

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The function \( f \) is defined by the power series
\[
f(x) = 1 + (x+1) + (x+1)^2 + \cdots + (x+1)^n + \cdots = \sum_{n=0}^{\infty} (x+1)^n
\]

a) Find the interval of convergence of the power series for \( f \). Justify your answer.
b) The power series above is the Taylor series for \( f \) about \( x = -1 \). Find the sum of the series for \( f \).
The function \( f \) is defined by the power series
\[
 f(x) = \frac{-x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n nx^n}{n+1}
\]
for all real numbers \( x \) for which the series converges.

The function \( g \) is defined by the power series
\[
 g(x) = \frac{-x}{2!} + \frac{2x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!}
\]
for all real numbers \( x \) for which the series converges.

a) Find the interval of convergence of the power series for \( f \). Justify your answer.

b) Find the interval of convergence of the power series for \( g \). Justify your answer.

14. The sum of the infinite geometric series
\[
 \frac{3}{2} + \frac{9}{16} + \frac{27}{128} + \frac{81}{1024} + \cdots
\]
is

(A) 1.60  (B) 2.35  (C) 2.40  (D) 2.45  (E) 2.50

6. What are all values of \( p \) for which
\[
 \int_1^\infty \frac{1}{x^p} \, dx
\]
converges?

A) \( p < -1 \)  B) \( p > 0 \)  C) \( p > \frac{1}{2} \)
D) \( p > 1 \)  E) There are no values of \( p \) for which this integral converges

4. Consider the series \( \sum_{n=1}^{\infty} \frac{e^n}{n!} \). If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

A) \( \lim_{n \to \infty} \frac{e^{n+1}}{n+1} < 1 \)  B) \( \lim_{n \to \infty} \frac{n!}{e} < 1 \)  C) \( \lim_{n \to \infty} \frac{n+1}{e} < 1 \)
D) \( \lim_{n \to \infty} \frac{e}{n+1} < 1 \)  E) \( \lim_{n \to \infty} \frac{e}{(n+1)!} < 1 \)
12. Which of the following series converges for all real numbers of \( x \)?

A) \( \sum_{n=1}^{\infty} \frac{x^n}{n} \)  
B) \( \sum_{n=1}^{\infty} \frac{x^n}{n^2} \)  
C) \( \sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}} \)  
D) \( \sum_{n=1}^{\infty} \frac{e^n x^n}{n!} \)  
E) \( \sum_{n=1}^{\infty} \frac{n! x^n}{e^n} \)

15. What are all values of \( x \) for which the series \( \sum_{n=1}^{\infty} \left( \frac{2}{x^2 + 1} \right)^n \) converges?

A) \(-1 < x < 1 \)  
B) \( x > 1 \) only  
C) \( x \geq 1 \) only  
D) \( x < -1 \) and \( x > 1 \) only  
E) \( x < -1 \) and \( x > 1 \) only  
F) \( x \leq -1 \) and \( x \geq 1 \) only  

79. Let \( f \) be a positive, continuous, decreasing function such that \( a_n = f(n) \).

If \( \sum_{n=1}^{\infty} a_n \) converges to \( k \), which of the following must be true?

A) \( \lim_{n \to \infty} a_n = k \)  
B) \( \int_1^n f(x) \, dx \) diverges  
C) \( \int_1^\infty f(x) \, dx \) diverges  
D) \( \int_1^\infty f(x) \, dx \) converges  
E) \( \int_1^\infty f(x) \, dx = k \)
82. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \leq a_n \leq b_n$ for all $n$, which of the following statements must be true?

A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges  
B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges  
C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges  
D) $\sum_{n=1}^{\infty} b_n$ converges  
E) $\sum_{n=1}^{\infty} b_n$ diverges

22. If $\lim_{b \to \infty} \int_{b-\infty}^{b} \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges  
(B) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges  
(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p+2}}$ converges  
(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges  
(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$  
II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$  
III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None  (B) II only  (C) III only  (D) I and II only  (E) I and III only
84. What are all values of $x$ for which the series $\sum_{n=1}^{\infty} \frac{(x + 2)^n}{\sqrt{n}}$ converges?  
   (A) $-3 < x < -1$  (B) $-3 \leq x < -1$  (C) $-3 \leq x \leq -1$  (D) $-1 \leq x < 1$  (E) $-1 \leq x \leq 1$

20. What are all values of $x$ for which the series $\sum_{n=1}^{\infty} \frac{(x - 2)^n}{n^3}$ converges?  
   (A) $-3 \leq x \leq 3$  (B) $-3 < x < 3$  (C) $-1 < x \leq 5$  (D) $-1 \leq x \leq 5$  (E) $-1 \leq x < 5$

24. Which of the following series diverge?  
   I. $\sum_{n=0}^{\infty} \frac{\sin 2}{\pi}$  
   II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  
   III. $\sum_{n=1}^{\infty} \frac{e^n}{e^n + 1}$  
   A) III only  B) I and II only  C) I and III only  
   D) II and III only  E) I, II, and III

22. What are all values of $p$ for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p + 1}$ converges?  
   A) $p > 0$  B) $p \geq 1$  C) $p > 1$  D) $p \geq 2$  E) $p > 2$