Volume using Cross Sections

2010 #4

Let R be the region in the first quadrant bounded by the graph of \( y = 2\sqrt{x} \), the horizontal line \( y = 6 \), and the y-axis, as shown in the figure.

Region R is the base of a solid. For each \( y \), where \( 0 \leq y \leq 6 \), the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

2010 Form B BC1

In the figure, R, is the shaded region in the first quadrant bounded by the graph \( y = 4\ln(3 - x) \), the horizontal line \( y = 6 \), and the vertical line \( x = 2 \).

c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.
2009 AB#4

Let R be the region in the first quadrant enclosed by the graphs of \( y = 2x \) and \( y = x^2 \), as shown in the figure.

![Graph of the region R](image)

a. The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area \( A(x) = \sin \left( \frac{\pi}{2} x \right) \). Find the volume of the solid.

b. Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

2011 BC Form B

The functions \( f \) and \( g \) are given by \( f(x) = \sqrt{x} \) and \( g(x) = 6 - x \). Let R be the region bounded by the x-axis and the graphs of \( f \) and \( g \), as shown in the figure above.

a) The region R is the base of a solid. For each \( y \), where \( 0 \leq y \leq 2 \), the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is \( 2y \). Write, but do not evaluate, an integral expression that gives the volume of the solid.
2009 Form B BC1

A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of \( y = f(x) \) for \( 0 \leq x \leq 30 \), where \( f(x) = 20 \sin \left( \frac{\pi x}{30} \right) \). Both x and y are measured in centimeters.

The region R is shown in the figure above. The derivative of \( f \) is \( f'(x) = \frac{2\pi}{3} \cos \left( \frac{\pi x}{30} \right) \).

a. The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

\[
\begin{align*}
\text{Region } R & \quad \text{in the figure above.}
\end{align*}
\]

2007 BC 1

Let R be the region in the first and second quadrants bounded above the graph of \( y = \frac{20}{1 + x^2} \) and below by the horizontal line \( y = 2 \).

a. The region R is the base of the solid. For this solid, the cross sections perpendicular to the x-axis are semi-circles. Find the volume of this solid.
2016 BC 5

The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by 
\[ r = \frac{1}{20} (3 + h^2), \]
where \( 0 \leq h \leq 10 \). The units of r and h are in inches.

b) Find the volume of the funnel.

2008 BC1

Let R be the region bounded by the graphs of \( y = \sin(x) \) and \( y = x^3 - 4x \), as shown in the figure.

b. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of a solid.

89. The region bounded by the graph of \( y = 2x - x^2 \), and the x-axis is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. What is the volume of the solid?

A) 1.333  B) 1.067  C) 0.577  D) 0.462  E) 0.267

87. Let R be the region in the first quadrant bounded above by the graph of \( y = \ln(3 - x) \), for \( 0 \leq x \leq 2 \), R is the base of a solid for which each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

A) 0.442  B) 1.029  C) 1.296  D) 3.233  E) 4.071