## Volume using Cross Sections

## 2010 \#4

Let R be the region in the first quadrant bounded by the graph of $\mathrm{y}=2 \sqrt{x}$, the horizontal line $\mathrm{y}=6$, and the $y$-axis, as shown in the figure.


Region R is the base of a solid. For each y , where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the $\mathbf{y}$-axis is a rectangle whose height is 3 times the length of its base in region R . Write, but do not evaluate, an integral expression that gives the volume of the solid.

## 2010 Form B BC1

In the figure, $R$, is the shaded region in the first quadrant bounded by the graph $y=4 \ln (3-x)$, the horizontal line $\mathrm{y}=6$, and the vertical line $\mathrm{x}=2$.
c. The region R is the base of a solid. For this solid, each cross section perpendicular to the $\mathbf{x}$-axis is a square. Find the volume of the solid.


Let $R$ be the region in the first quadrant enclosed by the graphs of $y=2 x$ and $y=x^{2}$, as shown in the figure.

a. The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the $\mathbf{x}$-axis has area $\mathrm{A}(\mathrm{x})=\sin \left(\frac{\pi}{2} x\right)$. Find the volume of the solid.
b. Another solid has the same base R. For this solid, the cross sections perpendicular to the $\mathbf{y}$-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

## 2011 BC Form B

The functions f and g are given by $f(x)=\sqrt{x}$ and $\mathrm{g}(\mathrm{x})=6-\mathrm{x}$. Let R be the region bounded by the $\mathrm{x}-$ axis and the graphs of f and g , as shown in the figure above.
a) The region R is the base of a solid. For each y , where $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the $\mathbf{y}$-axis is a rectangle whose base lies in R and whose height is 2 y . Write, but do not evaluate, an integral expression that gives the volume of the solid.


## 2009 Form B BC1

A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $\mathrm{y}=\mathrm{f}(\mathrm{x})$ for $0 \leq x \leq 30$, where $f(x)=20 \sin \left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f^{\prime}(x)=\frac{2 \pi}{3} \cos \left(\frac{\pi x}{30}\right)$.
a. The cake is a solid with base $R$. Cross sections of the cake perpendicular to the $\mathbf{x}$-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?


2007 BC 1
Let R be the region in the first and second quadrants bounded above the graph of $y=\frac{20}{1+x^{2}}$ and below by the horizontal line $\mathrm{y}=2$.
a. The region R is the base of the solid. For this solid, the cross sections perpendicular to the $\mathbf{x}$ axis are semi-circles. Find the volume of this solid.


The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r=\frac{1}{20}\left(3+h^{2}\right)$, where $0 \leq h \leq 10$. The units of r and h are in inches.
b) Find the volume of the funnel.

2008 BC1


Let $R$ be the region bounded by the graphs of $y=\sin (\pi x)$ and $y=x^{3}-4 x$, as shown in the figure.
b. The region R is the base of a solid. For this solid, each cross section perpendicular to the $\mathbf{x}$-axis is a square. Find the volume of a solid.
89. The region bounded by the graph of $y=2 x-x^{2}$, and the $x$-axis is the base of a solid. For this solid, each cross section perpendicular to the $\mathbf{x}$-axis is an equilateral triangle. What it the volume of the solid?
A) 1.333
B) 1.067
C) 0.577
D) 0.462
E) 0.267
87. Let R be the region in the first quadrant bounded above by the graph of $\mathrm{y}=\ln (3-\mathrm{x})$, for $0 \leq x \leq 2, \mathrm{R}$ is the base of a solid for which each cross section perpendicular to the $\mathbf{x}$-axis is a square. What is the volume of the solid?
A) 0.442
B) 1.029
C) 1.296
D) 3.233
E) 4.071

