

Using a definition of the derivative to find slope

A) Find the slope of $f(x) = x^2$ at the point (3,9)

B) Find the slope of $f(x) = \frac{2}{x}$ at $x = 4$

C) Find the slope of $f(x) = \frac{1}{x-4}$ at $x = 7$

$(7, \frac{1}{3})$
 \uparrow \uparrow
 a $f(a)$

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$f'(7) = \lim_{x \rightarrow 7} \frac{\frac{1}{x-4} - \frac{1}{3(x-4)}}{x-7}$$

$$\lim_{x \rightarrow 7} \frac{3 - (x-4)}{3(x-4)} = \frac{3 - x + 4}{3(x-4)} = \frac{7-x}{3(x-4)}$$

$$f'(7) = -\frac{1}{9}$$

D) Find the slope of $f(x) = 9 - x^2$ at the point (-3,0)

$$f'(-3) = \lim_{x \rightarrow -3} \frac{9 - x^2 - 0}{x + 3}$$

$$f'(-3) = 6$$

$$\lim_{x \rightarrow -3} \frac{(3+x)(3-x)}{x+3} = 6$$

$$\lim_{x \rightarrow 7} \frac{\frac{7-x}{1} - 1}{3(x-4)(x-7)}$$

$$\lim_{x \rightarrow 7} \frac{-1}{3(x-4)}$$

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What you'll Learn About

- Definition of the derivative
- Notation

Use the substitution $h = x - a$ to create the definition of the derivative

A₁) Set up a formula for the slope of $f(x) = x^2$ at $x = -1$

$$f'(-1) = \lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} \rightarrow \lim_{h \rightarrow 0} \frac{(h-1)^2 - 1}{h} = \frac{\Delta y}{\Delta x}$$

$h-1 \rightarrow -1$
 $\frac{+1}{+1} \quad h \rightarrow 0$

A₂) Use the substitution $h = x - a$ to set-up the definition of the derivative

$$f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

B₁) Set-up a formula for the slope of $f(x) = \frac{1}{x-2}$ at $x = 4$ $(4, \frac{1}{2})$

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \rightarrow 4} \frac{\frac{1}{x-2} - \frac{1}{2}}{x-4}$$

$$\Delta x = h = x - a$$

B₂) Use the substitution $h = x - a$ to set-up the definition of the derivative

$$\lim_{h \rightarrow 0} \frac{\frac{1}{(h+4)-2} - \frac{1}{2}}{h} = \frac{\frac{1}{h+2} - \frac{1}{2}}{h}$$

Given a definition of the derivative (slope) find the function that you are

$$h = x - a$$

$$h = x + 1$$

$$h - 1 = x$$

$$h = x - a$$

$$h = x - 4$$

$$h + 4 = x$$

taking the derivative of and the point you are finding the derivative(slope) at

A) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

B) $\lim_{x \rightarrow 2} \frac{\ln x - \ln 2}{x - 2}$

C) $\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$

D) $\lim_{h \rightarrow 0} \frac{\frac{2}{3+h} - \frac{2}{3}}{h}$

Another Definition: $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$