AP Problems Chapter 4

Mean Value Theorem

2013 BC3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), measured in ounces, are given in the table.

t(minutes)	0	1	2	3	4	5	6
C(t) ounces	0	5.3	8.8	11.2	12.8	13.8	14.5

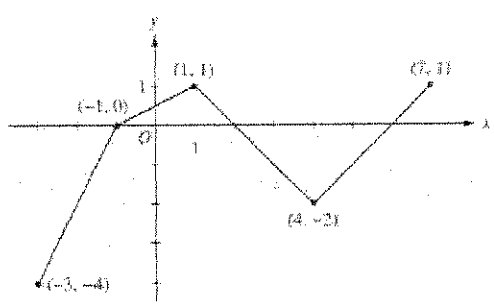
Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$. Justify your answer.

(2, 8.8)
(4, 12.8)

$m = \frac{12.8 - 8.8}{4-2} = \frac{4}{2} = 2$

Since C(t) is differentiable from [0, 6] the MVT guarantees a value between [2, 4) that the average rate of change equals C'(t).

Let g be a continuous function with g(2) = 5. The graph of the piecewise-linear function g', the derivative of g, is shown for $-3 \le x \le 7$.



Graph of g'

(-3, -4)
(7, 1)

Find the average rate of change of $g'(x)$, on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

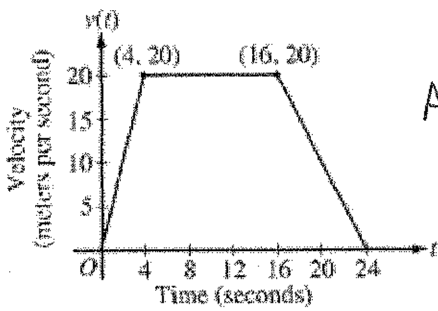
Average Rate of Change $= \frac{-4-1}{-3-7} = \frac{-5}{-10} = -\frac{1}{2}$

The MVT does not guarantee a value of c for $-3 < c < 7$ because $g'(x)$ is not differentiable at $x = -1, 1, 4$.

2005 AB5

A car is traveling on a straight road. For $8 \le t \le 24$ seconds, the car's velocity v(t), in meters per second, is modeled by the piecewise-linear function defined by the graph

(8,20) (24,0)



Avg Rate of Change $= \dfrac{20-0}{8-24} = \dfrac{20}{-16}$ m/sec^2

Find the average rate of change of v over the interval $8 \le t \le 24$. Does the Mean Value guarantee a value of c, for 8 < c < 24, such that $v'(t)$ is equal to this average rate of change? Why of why not?

The MVT does not guarantee a value of c for $8 < c < 24$ because $v'(t)$ is undefined at $t = 16$.

2004 BCB3

A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a ~~differential~~ differentiable function of t. Selected values of v(t) are shown.

t(min)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7	9.2	9.5	7	4.5	2.4	2.4	4.3	7.3

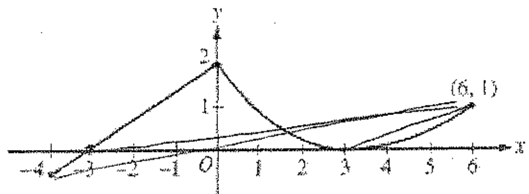
Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval 0 < t < 40? Justify your answer

2 because the ~~slope~~ avg velocity between [0,15] and [25,30] is zero. ~~The MVT guarantees~~

Since the function v is differentiable from (0,40) the MVT guarantees the average velocity will equal the instantaneous velocity.

2009 BC3

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at x = 3, as shown in the figure above. On the interval 0 < x < 6, the function f is twice differentiable, with $f''(x) > 0$.



Graph of f

Is there a value a, , for which the Mean Value Theorem, applied to the interval [a, 6], guarantees a value c, a < c < 6, at which $f'(c) = \frac{1}{3}$? Justify your answer.

Since f is differentiable, the MVT guarantees that the average ~~differen~~ rate of change will equal f'(c) on the given interval.

(6, 1)

~~(3, 0)~~ (3, 0)

slope = $\frac{1}{3}$

[3, 6]

a

2011 BCB5

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position of the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table gives values of B(t) and Ben's velocity, v(t), measured in meters per second, at selected times t.

t(seconds)	0	10	40	60
B(t) (meters)	100	136	9	49
V(t) meters per second	2	2.3	2.5	4.6

For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

Since B(t) is differentiable the MVT guarantees the Avg Rate of Change will equal the instantaneous rate of change on the interval $40 < t < 60$

$m = \frac{49-9}{60-40}$

$m = \frac{40}{20}$

$m = 2$

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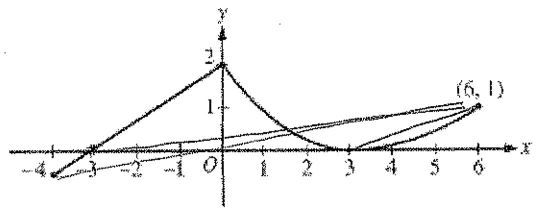

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$m = \frac{49-9}{60-40}$

$m = \frac{40}{20}$

$m = 2$

3. (calculator not allowed)
Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0, 3]$?

~~(A) 0 only~~
~~(B) 2 only~~
(C) 3 only
(D) 0 and 3
(E) 2 and 3

$(0,0)\ (3,0)$

$m = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

Not in interval

8. (calculator not allowed)
The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between (0, 0) and (4, 2). What are the coordinates of this point?

~~(A) (2, 1)~~
(B) (1, 1)
~~(C) $(2, \sqrt{2})$~~
(D) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$
(E) None of the above

$m = \frac{2}{4} = \frac{1}{2}$

$y' = \frac{1}{2\sqrt{x}}$

$\frac{1}{2} = \frac{1}{2\sqrt{x}}$

$2\sqrt{x} = 2$

$\sqrt{x} = 1$

$x = 1$